

The Smile of the Mathematicians

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Some mathematicians have a problem with a smile – a mathematical one. The talk will present results of a joint project with surgeons in the field of cranio-maxillo-facial surgery. In this kind of surgery, faces are operatively changed to get rid of unpleasant face distortions. Bone from the upper or lower jaws is removed or shifted up to cm's. The questions treated by mathematicians are:

1. operation planning within teleconferences of ZIB and the clinic, based on a detailed geometric model of skull and facial tissue,
2. prediction of the facial appearance on the basis of numerical simulation of the patients soft tissue, based on fast adaptive multilevel finite element methods for the elastomechanic partial differential equations, either the linear Navier-Lam equations or nonlinear enrichments like geometric nonlinearity or nonlinear material laws of Ogden type.

The second problem type leads to nonconvex optimization. Extensions of affine conjugate Newton methods for convex optimization problems to the nonconvex case here (more precisely: polyconvex case) are given. During the talk, a lot of results for real patients are inserted, including the effect of such operations on the smile.