A Variational Approach and Optimization for Control Problems of Elastic Body Motion

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Initial-boundary problems in the linear theory of elasticity is considered. Based on the integral formulation of stress-strain relations a new dynamical variational principle, in which displacement, stress, and momentum functions are varied, is proposed and discussed. To minimize the nonnegative functional under initial, boundary, and equilibrium constraints a finite element algorithm for spline approximation of the unknown functions is worked out. The algorithm gives one the possibility to estimate explicitly the local and integral quality of numerical solutions obtained. An effective numerical approach to optimal control problems with quadratic objective functionals is developed for elastic body motion. As an example, a 3D problem of optimal motion of a rectilinear elastic prism are considered for the terminal total mechanical energy to be minimized. The numerical results and their error estimates are presented and discussed.