Nonlinear Chance-Constrained Optimization and Applications

M. Paffrath and B. Weber and U. Wever

Siemens AG, Corporate Technology, Otto-Hahn-Ring 6, D81730 Munich, Germany,
meinhard.paffrath@siemens.com

In practical applications, optimization of structures has to deal with fluctuating parameters, scattering environment data (e.g. temperatures), uncertain material and geometry parameters. Under these prerequisites, a deterministic optimization may not be adequate but a stochastic optimization should be performed instead. Let $\vec{X} = (X_1, .., X_n)^T$ be a vector of independent stochastic input variables X_i with mean μ_i, standard deviation σ_i and distribution $D_i(\mu_i, \sigma_i)$, $\vec{\mu} = (\mu_1, ..., \mu_n)^T$, $\vec{\sigma} = (\sigma_1, ..., \sigma_n)^T$. A general chance-constrained optimization problem may be stated as follows

$$\begin{aligned} &\text{Minimize } S(f, \vec{\mu}, \vec{\sigma}) \\ &\text{w.r.t.} \\ &P(g_i, \vec{\mu}, \vec{\sigma}) \leq \epsilon_i, \qquad i = 1, .., m \end{aligned} \tag{1}$$

with design parameters $\vec{\mu}, \vec{\sigma}$, linear or nonlinear functions

$$\begin{aligned} f \quad &: \quad \mathbb{R}^n \to \mathbb{R} \\ &\quad\ \vec{x} \mapsto f(\vec{x}) \\ g_i \quad &: \quad \mathbb{R}^n \to \mathbb{R} \\ &\quad\ \vec{x} \mapsto g_i(\vec{x}) \qquad i = 1, .., m \end{aligned}$$

and failure probabilities

$$P(g_i, \vec{\mu}, \vec{\sigma}) = \int_{g_i(\vec{x}) \leq 0} \rho(\vec{x}, \vec{\mu}, \vec{\sigma}) d\vec{x} \qquad i = 1, .., m \tag{2}$$

$\vec{x}$ denotes the vector of input parameters (realizations of $\vec{X}$), and ρ the joint probability density function of $\vec{X}$. Here we restrict ourselves to probability distributions with compact support $\Omega(\vec{\mu}, \vec{\sigma})$ (a cuboid) which are the most important distributions in practical applications. The operator $S(f, \vec{\mu}, \vec{\sigma})$ in the objective function may be a stochastic moment of f or again a failure probability. So e.g. minimization of mean and variance of f is given by:

$$\text{Minimize } \bar{f} := E(f, \vec{\mu}, \vec{\sigma}) = \int_{\Omega(\vec{\mu}, \vec{\sigma})} f(\vec{x}) \rho(\vec{x}, \vec{\mu}, \vec{\sigma}) d\vec{x} \tag{3}$$

$$\text{Minimize } Var(f, \vec{\mu}, \vec{\sigma}) = \int_{\Omega(\vec{\mu}, \vec{\sigma})} (\bar{f} - f(\vec{x}))^2 \rho(\vec{x}, \vec{\mu}, \vec{\sigma}) d\vec{x} \tag{4}$$

Numerically critical is the solution of the multi-dimensional integral in (2) for it generally requires the use of computationally expensive sampling methods. Therefore we propose, as first step of the solution procedure of (1), the solution of an easier manageable problem where the chance constraints are replaced by absolute reliability constraints:

$$P(g_i, \vec{\mu}, \vec{\sigma}) = 0, \qquad i = 1, .., m$$

which is equivalent to

$$\min_{\vec{x} \in \Omega(\vec{\mu}, \vec{\sigma})} g_i(\vec{x}) \geq 0 \quad i = 1, .., m$$

Now the optimization problem has the form

$$\begin{aligned} &\text{Minimize } S(f, \vec{\mu}, \vec{\sigma}) \\ &\text{w.r.t.} \\ &\min_{\vec{x} \in \Omega(\vec{\mu}, \vec{\sigma})} g_i(\vec{x}) \geq 0 \quad i = 1, .., m \end{aligned} \tag{5}$$

In our presentation, we will concentrate on efficient numerical methods for evaluation of the integrals in (3), (4), the solution of (5) and some applications in the field of mathematical engineering.